14.3 ~ Partial Derivatives

* the variable you are taking the derivative with respect to is the one you care about, every other variable is constant!

skip Thm

~~Clairaut's theorem~~

~~f is defined on domain D.~~

~~f_{xy} & f_{yx} both continuous $\Longrightarrow f_{xy} = f_{yx}$~~

(30)

Ex: $f(x,y,z) = x\sin(y-z)$ Find f_x, f_y, f_z

$f_x = \sin(y-z)$ since y & z are constant, $\sin(y-z)$ is constant

$f_y = x\cos(y-z)\cdot(1) \Rightarrow f_y = x\cos(y-z)$

sine was operating on y so we had to differentiate

chain rule: $\frac{d}{dy}(y-z) = 1-0 = 1$

$f_z = x\cos(y-z)\cdot(-1) \Rightarrow f_z = -x\cos(y-z)$

similar arguement as above but in terms of z.

chain rule: $\frac{d}{dz}(y-z) = 0-1 = -1$

(32)

Ex: $w = ze^{xyz}$ Find $\frac{dw}{dx}, \frac{dw}{dy}, \frac{dw}{dz}$

*recall $\frac{d}{du}[e^u] = u'e^u$

$\frac{dw}{dx} = z\cdot\frac{d}{dx}(xyz)\cdot e^{xyz} \Rightarrow \frac{dw}{dx} = yz^2e^{xyz}$

$\frac{dw}{dy} = z\,\frac{d}{dy}(xyz)\cdot e^{xyz} \Rightarrow \frac{dw}{dy} = xz^2e^{xyz}$

$\frac{dw}{dz} = \frac{d}{dz}(z)\cdot e^{xyz} + z\,\frac{d}{dz}(e^{xyz})$ ⟵ product rule! You have 2 terms multiplied together that each involve the variable z

$= e^{xyz} + z\,\frac{d}{dz}(xyz)e^{xyz}$

$= e^{xyz} + xyze^{xyz}$

$\frac{dw}{dz} = (1+xyz)e^{xyz}$

31 on steroids

Ex: $f = x\ln(xy^2 + 2y^3z + 3e^xz)$ Find f_x, f_y, f_z

$f_x = \frac{d}{dx}(x)\ln(xy^2+2y^3z+3e^xz) + x\,\frac{d}{dx}\left(\ln(xy^2+2y^3z+3e^xz)\right)$

$= \ln(xy^2+2y^3z+3e^xz) + x\left(\frac{1}{xy^2+2y^3z+3e^xz}\right)\frac{d}{dx}(xy^2+2y^3z+3e^xz)$

$$= \ln(xy^2+2y^3z+3e^xz) + \frac{x}{xy^2+2y^3z+3e^xz}(y^2+0+3e^xz)$$

$$f_x = \ln(xy^2+2y^3z+3e^xz) + \frac{xy^2+3xe^xz}{xy^2+2y^3z+3e^xz}$$

$$f_y = x\frac{d}{dy}(\ln(xy^2+2y^3z+3e^xz))$$

$$= x\left(\frac{1}{xy^2+2y^3z+3e^xz}\right)\cdot\frac{d}{dy}(xy^2+2y^3z+3e^xz)$$

$$= x\left(\frac{1}{xy^2+2y^3z+3e^xz}\right)(2xy+6y^2z+0)$$

$$f_y = \frac{2x^2y+6xy^2z}{xy^2+2y^3z+3e^xz}$$

$$f_z = x\frac{d}{dz}(\ln(xy^2+2y^3z+3e^xz))$$

$$= x\left(\frac{1}{xy^2+2y^3z+3e^xz}\right)\cdot\frac{d}{dz}(xy^2+2y^3z+3e^xz)$$

$$= x\left(\frac{1}{xy^2+2y^3z+3e^xz}\right)(0+2y^3+3e^x)$$

$$f_z = \frac{2xy^3+3xe^x}{xy^2+2y^3z+3e^xz}$$

---

* checkout the LIMIT LAWS p. 99

7.8 ~ Improper Integrals

What makes an integral improper?

① infinite integrals $\Rightarrow \int_a^\infty, \int_{-\infty}^b, \int_{-\infty}^\infty$

② discontinuous integrands $\Rightarrow \int_0^3 \ln x\,dx, \int_{-1}^1 \frac{1}{x+1/2}dx$

*recall: $\int \frac{1}{x^p}dx$ converges if $p>1$, diverges if $p \le 1$ } same for a series $\Sigma \frac{1}{n^p}$

type 1

14 Ex: $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx \Rightarrow u = -\sqrt{x} \Rightarrow du = -\frac{1}{2\sqrt{x}}dx \Rightarrow \frac{1}{\sqrt{x}}dx = -2du$

$$= \int -2e^u du = \lim_{b\to\infty}[-2e^{-\sqrt{x}}]_1^b$$

$$= \lim_{b\to\infty} -2(e^{-\sqrt{b}} - e^{-1}) = -2(\cancel{e^{-\infty}}^{\,0} - e^{-1}) = 2e^{-1}, \text{ convergent}$$

type 1

25 Ex: $\int_e^\infty \frac{1}{x(\ln x)^3}dx \Rightarrow u = \ln x, \Rightarrow du = \frac{1}{x}dx$

$$\Rightarrow \int \frac{1}{u^3}du = -\frac{1}{2u^2}$$

$$\Rightarrow -\frac{1}{2}\lim_{b\to\infty}\left[\frac{1}{(\ln x)^2}\right]_e^b = -\frac{1}{2}\left[\lim_{b\to\infty}\left(\frac{1}{(\ln b)^2}\right) - \frac{1}{(\ln e)^2}\right]$$

$$= -\frac{1}{2}\left[\frac{1}{(\ln\infty)^2} - \frac{1}{1^2}\right] = -\frac{1}{2}\left(\cancel{\frac{1}{\infty^2}}^{\,0} - 1\right) = \frac{1}{2}, \text{ convergent}$$

type 2

25 modified

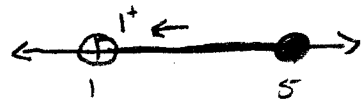
Ex: $\int_1^5 \frac{1}{x(\ln x)^3}\,dx \Rightarrow u = \cdots, \; du = \cdots$

$\Rightarrow -\frac{1}{2u^2}$, like before

$\Rightarrow -\frac{1}{2} \lim_{a\to 1^+} \left[\frac{1}{(\ln x)^2}\right]_a^5 = -\frac{1}{2}\left[\frac{1}{(\ln 5)^2} - \lim_{a\to 1^+} \frac{1}{(\ln a)^2}\right]$

$= -\frac{1}{2}\left[\frac{1}{(\ln 5)^2} - \frac{1}{0^2}\right] = -\frac{1}{2}[\# - \infty]$

$= \infty$, diverges



11.1 ~ Sequences

$\lim_{n\to\infty} a_n = L$

* L, finite sequence converges
* L, infinite sequence diverges

for all

$a_{n+1} > a_n \; \forall n \geq 1$ sequence is increasing

$a_{n+1} < a_n \; \forall n \geq 1$ sequence is decreasing

18

Ex: $a_n = \frac{n^3}{n^3+1}$, converge or diverge?

$\lim_{n\to\infty} \frac{n^3}{n^3+1}\left(\frac{1/n^3}{1/n^3}\right) = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n^3}} = \frac{1}{1+\frac{1}{\infty}} = \frac{1}{1+0} = 1$ converges

24

Ex: try for fun, answer is $\frac{1}{3}$ converges.

62

Ex: Inc, Dec, Neither. Is it bounded?

$a_n = \frac{2n-3}{3n+4}$, let $f(x) = \frac{2x-3}{3x+4}$

$f' = \frac{2(3x+4) - 3(2x-3)}{(3x+4)^2} = \frac{\cancel{6x}+8-\cancel{6x}+9}{(3x+4)^2}$

$f' = \frac{17}{(3x+4)^2}$ always positive $\Rightarrow$ increasing

$a_1 = \frac{2-3}{3+4} = -\frac{1}{7}$ $\quad$ a_n as $n\to\infty$ is $\lim_{n\to\infty} \frac{2n-3}{3n+4} = \frac{2}{3}$

$\Rightarrow$ bounded $-\frac{1}{7} \leq a_n < \frac{2}{3}$

64

Ex: Inc, Dec, Niether, bounded?

$a_n = n e^{-n} \Rightarrow f(x) = x e^{-x}$

$f' = e^{-x} - x e^{-x} = (1-x) e^{-x}$

e^{-x} is always positive

$(1-x)$ is negative for $x > 1$

$\Big\} f' < 0, \ x > 1$

$\Rightarrow$ a_n is a positive decreasing sequence

$\Rightarrow a_1 > a_2 > \cdots > a_n > 0, \ a_1 = 1e^{-1} = e^{-1}$

$\Rightarrow$ bounded $0 < a_n \le \frac{1}{e}$

60

Ex: Inc, Dec, Niether, bounded?

$a_n = (-2)^{n+1} \Rightarrow$ alternating sign $\Rightarrow$ niether (not monotonic)

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^{n+1}(2)^{n+1} = \pm 2^{\infty} = \pm\infty \Rightarrow$ not bounded

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11.2 – 11.8 ~Series *skip 11.4, 11.7

series converges $\Longrightarrow \lim_{n\to\infty} a_n = 0$

- Divergence Test $\lim_{n\to\infty} a_n \ne 0 \Rightarrow$ diverges
- Integral Test
  - decreasing
  - positive
  - continuous

  $[x_0, \infty) \Rightarrow \left[\sum a_n \text{ converges/diverges} \Longleftrightarrow \int_{x_0}^{\infty} f(x)\,dx \text{ exists/DNE} \right]$
- p-series $\sum \frac{1}{n^p}$ converges $p > 1$, diverges $p \le 1$
- alternating series test $\sum_{n=0}^{\infty} (-1)^n b_n, \ b_n > 0$ ① $\lim_{n\to\infty} b_n = 0$ ② $b_{n+1} \le b_n$ $\Rightarrow$ converges
- Ratio Test $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$
  - $L < 1 \Rightarrow$ abs. converges
  - $L > 1 \Rightarrow$ diverges
  - $L = 1 \Rightarrow$ inconclusive

*skip n^{th}-root

Power Series $\sum_{n=0}^{\infty} c_n (x-a)^n$

- converges $x = a$
- converges $\forall x$
- converges $|x-a| < R$
  diverges $|x-a| > R$

* R is radius of convergence
* I is interval of convergence (x values for which series converges)

Ex's: Find radius of convergence (R) and interval of convergence (I).

(6) $\sum_{n=1}^{\infty} \sqrt{n}\, x^n$

*always start with ratio test

$$\lim_{n\to\infty} \left| \frac{\sqrt{n+1}\, x^{n+1}}{\sqrt{n}\, x^n} \right| = \lim_{n\to\infty} \left| x \sqrt{\frac{n+1}{n}} \right|$$

$$= \lim_{n\to\infty} |x| \left| \sqrt{\frac{n+1}{n}} \right| = |x| \lim_{n\to\infty} \sqrt{\frac{n+1}{n}}$$

$$= |x| \sqrt{\lim_{n\to\infty} \frac{n+1}{n}} = |x| < 1$$

* force result to be less than 1 for absolute convergence

$\Rightarrow$ $R = 1$

so $-1 \boxed{\substack{< \\ \text{or} \\ \le}} x \boxed{\substack{< \\ \text{or} \\ \le}} 1$?

* Test the interval end pts.

Let $x = -1$

$\Rightarrow \sum_{n=1}^{\infty} \sqrt{n}\, x^n = \sum_{n=1}^{\infty} \sqrt{n}\, (-1)^n$, This is an alternating series

where $b_n = \sqrt{n} > 0$ ✓

(1) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \sqrt{n} = \infty \neq 0 \Rightarrow$ diverges

$\Rightarrow -1 < x$ (diverges means $<$ or $>$ only)

Let $x = 1$

$\Rightarrow \sum_{n=1}^{\infty} \sqrt{n}\, x^n = \sum_{n=1}^{\infty} \sqrt{n}$, Try Divergence Test

$\lim_{n\to\infty} \sqrt{n} = \sqrt{\infty} = \infty \neq 0 \Rightarrow$ Diverges

$\Rightarrow x < 1$

Interval of Convergence: $I = (-1, 1)$ or $-1 < x < 1$

(14) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

Ratio Test $\quad$ *note $\left|\frac{(-1)^{n+1}}{(-1)^n}\right| = 1$

$$\lim_{n\to\infty} \left| \frac{x^{2(n+1)}}{(2(n+1))!} \cdot \frac{(2n)!}{x^{2n}} \right| = \lim_{n\to\infty} \left| \frac{x^{2n+2}}{x^{2n}} \cdot \frac{(2n)!}{(2n+2)!} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x^2 \cdot \cancel{(2n)!}}{(2n+2)(2n+1)\cancel{(2n)!}} \right| = x^2 \lim_{n\to\infty} \left| \frac{1}{(2n+2)(2n+1)} \right|$$

$= x^2\left(\frac{1}{\infty}\right) = x^2(0) = 0 < 1$ always ; i.e. it always converges

$\Rightarrow$ $R = \infty$ $\Rightarrow$ $I = (-\infty, \infty)$

Radius of Convergence $\quad$ Interval of convergence

(26) $\sum_{n=2}^{\infty} \frac{x^{2n}}{n(\ln n)^2}$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{x^{2(n+1)}}{(n+1)(\ln(n+1))^2} \cdot \frac{n(\ln n)^2}{x^{2n}} \right| = \lim_{n\to\infty} \left| x^2 \left(\frac{n}{n+1}\right) \frac{(\ln n)^2}{(\ln(n+1))^2} \right|$$

$$= x^2 \lim_{n\to\infty}\left(\frac{n}{n+1}\right) \cdot \lim_{n\to\infty} \frac{(\ln n)^2}{(\ln(n+1))^2} = x^2 \left(\frac{\infty}{\infty}\right)\left(\frac{\infty}{\infty}\right)$$

good trash! use L'Hospitals Rule

$$= x^2 \cdot \lim_{n\to\infty}\left(\frac{1}{1}\right) \cdot \lim_{n\to\infty} \frac{\cancel{2}(\ln n)\cdot\frac{1}{n}}{\cancel{2}(\ln(n+1))\left(\frac{1}{n+1}\right)} = x^2 \lim_{n\to\infty} \frac{\ln n}{\ln(n+1)} \cdot \cancel{\lim_{n\to\infty} \frac{n+1}{n}}^{\,1}$$

$$= x^2 \left(\frac{\infty}{\infty}\right)\left(\frac{\infty}{\infty}\right) = x^2 \lim_{n\to\infty} \frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{n+1}\right)} = x^2 \cancel{\lim_{n\to\infty} \frac{n+1}{n}}^{\,1}$$

$= x^2 < 1 \Rightarrow |x| < 1$ because $\sqrt{x^2} < 1$ is $x \lessgtr {}^{1}_{-1}$

$\Rightarrow$ $R = 1$ so $-1 \;\boxed{\begin{smallmatrix} < \\ \text{or} \\ \le \end{smallmatrix}}\; x \;\boxed{\begin{smallmatrix} < \\ \text{or} \\ \le \end{smallmatrix}}\; 1$?

Testing the interval end pts

$x = \pm 1 \Rightarrow \sum_{n=2}^{\infty} \frac{(\pm1)^{2n}}{n(\ln n)^2} = \sum_{n=2}^{\infty} \frac{((\pm1)^2)^n}{n(\ln n)^2} = \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

Divergence Test $\Rightarrow \lim_{n\to\infty} \frac{1}{n(\ln n)^2} = 0 \Rightarrow$ inconclusive

Integral Test $\Rightarrow$ need f to be positive, continuous, & decreasing

- $f(x) = \frac{1}{x(\ln x)^2} > 0$ on $[2, \infty)$
- f is cont. on $[2,\infty)$, i.e. denominator is non zero

- $f' = \dfrac{-\left((\ln x)^2 + 2x(\ln x)\frac{1}{x}\right)}{[x(\ln x)^2]^2} = -\dfrac{\cancel{(\ln x)}(\ln x + 2)}{x^2(\ln x)^{\cancel{4}3}}$

$= -\dfrac{\ln x + 2}{x^2(\ln x)^3} < 0$ on $[2, \infty)$ since

$\ln x \geq \ln 2 > 0$ and $x^2 > 0$ for $x \geq 2$

Therefore f is decreasing on $[2, \infty)$

Since those conditions are satisfied we can go on with the Integral Test.

$\int_2^\infty \frac{1}{x(\ln x)^2}\,dx$, $u = \ln x$, $du = \frac{1}{x}dx$

$\Rightarrow \int \frac{1}{u^2}\,du = -\frac{1}{u}$

$\Rightarrow \lim_{b\to\infty}\left[-\frac{1}{\ln x}\right]_2^b = -\lim_{b\to\infty}\frac{1}{\ln b} + \frac{1}{\ln 2} = \frac{1}{\ln\infty} + \frac{1}{\ln 2} = \frac{1}{\infty} + \frac{1}{\ln 2}$

$= 0 + \frac{1}{\ln 2} = \frac{1}{\ln 2}$, finite $\Rightarrow$ converges

$\Rightarrow$ the series $\sum_{n=2}^\infty \frac{1}{n(\ln n)^2}$ converges

$\Rightarrow \sum_{n=2}^\infty \frac{x^{2n}}{n(\ln n)^2}$ converges at $x = \pm 1$

$\Rightarrow -1 \leq x \leq 1$

Interval of Convergence : $I = [-1, 1]$

* for more practice try other problems from 11.8 numbers 3 - 28

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* skip 11.9 and 11.11

11.10 ~ Taylor and Maclaurin Series

from 11.9

*recall: power series $\sum_{n=0}^\infty c_n(x-a)^n$

If the power series has a radius of convergence then the function f defined by $f(x) = \sum_{n=0}^\infty c_n(x-a)^n$ is differentiable / integrable / continuous on the interval $(a-R, a+R)$

$\Rightarrow c_n = \dfrac{f^{(n)}(a)}{n!}$ in 11.10

Taylor Series: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$, where f is centered @ a

$\Rightarrow f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$

Maclaurin Series: (let $a=0$) $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

$\Rightarrow f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$

* NOTE: a radius of convergence is <u>always</u> associated with a function's series representation

Ex's:

Find Maclaurin series for each function.

(8) $f(x) = \cos(3x)$

n	$f^{(n)}(x)$	$\frac{f^{(n)}(a)}{n!}(x-a)^n = \frac{f^{(n)}(0)}{n!}x^n$
0	$f^{(0)} = f = \cos 3x$	$\frac{1}{0!}x^0 = \frac{(-1)^0 3^0}{0!}x^0$
1	$f^{(1)} = f' = -3\sin 3x$	$\frac{0}{1!}x^1 = 0$
2	$f^{(2)} = f'' = -3^2\cos 3x$	$-\frac{3^2}{2!}x^2 = \frac{(-1)^1 3^{2(1)}}{(2(1))!}x^{2(1)}$
3	$f^{(3)} = f''' = 3^3 \sin 3x$	$\frac{0}{3!}x^3 = 0$
4	$f^{(4)} = f'''' = 3^4 \cos 3x$	$\frac{3^4}{4!}x^4 = \frac{(-1)^2 3^{2(2)}}{(2(2))!}x^{2(2)}$
⋮	⋮	⋮

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{(2n)!} x^{2n}, \text{ for all } x}$$

<u>Ratio Test</u>

$$\lim_{n\to\infty}\left| \frac{3^{2(n+1)} \cdot x^{2(n+1)}}{(2(n+1))!} \cdot \frac{(2n)!}{3^{2n} x^{2n}} \right|$$

$$= \lim_{n\to\infty}\left| \frac{3^{2n+2} x^{2n+2} (2n)!}{\cancel{3^{2n}}\,\cancel{x^{2n}}\,(2n+2)!} \right| = \lim_{n\to\infty}\left| \frac{3^2 x^2 \cancel{(2n)!}}{(2n+2)(2n+1)\cancel{(2n)!}} \right| = 3^2x^2 \lim_{n\to\infty}\left|\frac{1}{(2n+2)(2n+1)}\right|$$

$= 0 < 1$ always $\Rightarrow R = \infty \Rightarrow I = (-\infty, \infty)$, all real numbers

* using the above series representation find $\sin(3x)$

$\frac{d}{dx}[\cos(3x)] = -3\sin(3x) \Rightarrow \sin(3x) = -\frac{1}{3}\cdot\frac{d}{dx}[\cos(3x)]$

$$\Rightarrow \sin(3x) = -\frac{1}{3}\cdot\frac{d}{dx}\left[\sum_{n=0}^{\infty}\frac{(-1)^n 3^{2n}}{(2n)!}x^{2n}\right] = -\frac{1}{3}\sum_{n=1}^{\infty}\frac{(-1)^n 3^{2n}}{(2n)!}\cdot\frac{d}{dx}\left[x^{2n}\right]$$

$$= -\frac{1}{3}\sum_{n=1}^{\infty}\frac{(-1)^n 3^{2n}}{(2n)!}\cdot(2n)x^{2n-1} = -\frac{1}{3}\sum_{n=1}^{\infty}\frac{(-1)^n 3^{2n}}{(2n-1)!}x^{2n-1}$$

$$\boxed{g(x) = \sin(3x) = -\frac{1}{3}\sum_{n=1}^{\infty} \frac{(-1)^n 3^{2n}}{(2n-1)!} x^{2n-1}, \text{ for all } x}$$

This can be rewrite in a more standard way if you like!

$$-\frac{1}{3}\sum_{n=1}^{\infty} \frac{(-1)^n 3^{2n}}{(2n-1)!} x^{2n-1} = \sum_{n=1}^{\infty} \overbrace{(-1)(3^{-1})}^{-\frac{1}{3}} \frac{(-1)^n 3^{2n}}{(2n-1)!} x^{2n-1}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3^{2n-1}}{(2n-1)!} x^{2n-1} = \sum_{n=0}^{\infty} \frac{(-1)^{(n+1)+1} 3^{2(n+1)-1}}{(2(n+1)-1)!} x^{2(n+1)-1}$$

in standard form

$$\boxed{g(x) = \sin(3x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1}}{(2n+1)!} x^{2n+1}, \text{ for all } x}$$

* bring the const. into the sum

* change lower index from $n=1$ to $n=0$ in the sum.

*note: $(-1)^{n+2} = (-1)^n$

$n=0 \to (-1)^2 = (-1)^0 = 1$

$n=1 \to (-1)^3 = (-1)^1 = -1$

$n=2 \to (-1)^4 = (-1)^2 = 1$

$\vdots$

You can compare how $\sin(3x)$ and $\cos(3x)$ looks to $\sin x$ and $\cos x$ derived in 11.10 p. 740

(10) $f(x) = xe^x$

n	$f^{(n)}(x)$	$\frac{f^{(n)}(0)}{n!}x^n$
0	xe^x	$\frac{0}{0!}x^0 = 0$
1	$e^x + xe^x$	$\frac{1}{1!}x^1 = \frac{1}{0!}x^1$
2	$2e^x + xe^x$	$\frac{2}{2!}x^2 = \frac{1}{1!}x^2$
3	$3e^x + xe^x$	$\frac{3}{3!}x^3 = \frac{1}{2!}x^3$
$\vdots$		$\vdots$

$$\frac{n}{n!}x^n = \frac{\cancel{n}}{\cancel{n}(n-1)!}x^n$$

$$\boxed{f(x) = \sum_{n=1}^{\infty} \frac{1}{(n-1)!} x^n, \text{ for all } x}$$

in standard form where $n = 0, 1, 2$

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} x^{n+1}, \text{ for all } x}$$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{\cancel{x^{n}}^{\,n+1}}{((n+1)-1)!} \cdot \frac{(n-1)!}{\cancel{x^n}}\right| = \lim_{n\to\infty}\left|\frac{x\,(n-1)!}{n!}\right| = |x| \lim_{n\to\infty} \frac{\cancel{(n-1)!}}{n\cancel{(n-1)!}}$$

$$= |x| \lim_{n\to\infty} \frac{1}{n} = |x|(0) = 0 < 1 \Rightarrow R = \infty \Rightarrow I = (-\infty, \infty)$$

*compare this solution to x multiplied by the definition of e^x on p. 743 table 1

* practice more Maclaurin Series on p. 746 #s 5-10.