Practice / Test Problems

1. Show that the cylinder is a regular surface and find parametrizations whose coordinate nhbds cover it.
$$\text{Cyl.} = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2=1\}$$

2. Is the set $\{(x,y,z) \in \mathbb{R}^3 \mid z=0,\ x^2+y^2 \le 1\}$ a reg. surf?
Is the set $\{$ " $\mid$ " $,\ x^2+y^2 < 1\}$ a reg. surf?

3. Show that the two-sheeted cone, $\{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2-z^2=0\}$, is <u>not</u> a regular surface.

4. Let $f(x,y,z) = z^2$. Prove that zero (0) is not a regular value of f, and yet $f^{-1}(0)$ is a regular surface.

5. Let $P = \{(x,y,z) \in \mathbb{R}^3 \mid x=y\}$ and let $\mathbf{x} : U \subset \mathbb{R}^2 \to \mathbb{R}^3$ be $\mathbf{x}(u,v) = (u+v,\ u+v,\ uv)$, where $U = \{(u,v) \in \mathbb{R}^2 \mid u > v\}$. Clearly $\mathbf{x}(U) \subset P$. Is $\mathbf{x}$ a parametrization of P?

Homework Problem

4/2/06

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a. Show that $\mathbf{X} : U \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ given by:

$$\mathbf{X}(u,v) = (a \sin(u) \cos(v),\ b \sin(u) \sin(v),\ c \cos(u))$$

$a, b, c \neq 0$ $\qquad$ $0 < u < \pi$

$a, b, c \in \mathbb{R}$ $\qquad$ $0 < v < 2\pi$

is a parametrization for the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

b. Describe geometrically the curves $u =$ constant on the ellipsoid.

Solutions to Practice/Test Problems

Problem
1. Show that the cylinder is a regular surface

$$C = Cyl = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1\}$$

Use proposition 1.

If $f: U \to \mathbb{R}$ is differentiable in an open set $U \subset \mathbb{R}^2$, then the graph of f, $(x,y,f(x,y))$ for $(x,y) \in U$ is a regular surface.

Let $X_1 = (x, \sqrt{1-x^2}, z)$ $\qquad U = \{(x,z) \in \mathbb{R}^2 \mid x^2 < 1\}$

$f = \sqrt{1-x^2}$

$\frac{\partial f}{\partial x}$ is differentiable $\forall\ (x,z) \in U$

$\frac{\partial f}{\partial z} = 0 \quad \forall\ (x,z) \in U$

$\therefore$ The open ½ cylinder is a regular surface

We can easily cover C with 4 similar parametrizations, so C is a regular surface

$X_2 = (x, -\sqrt{1-x^2}, z)$

$X_3 = (\sqrt{1-y^2}, y, z)$

$X_4 = (-\sqrt{1-y^2}, y, z)$

Poblem 2

Is the set $\{(x,y,z) \in \mathbb{R}^3 \mid z=0,\ x^2+y^2 \leq 1\}$ a reg. surf?

Let $f = f(x,y) = z = 0$

$U = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1\}$

f is differentiable, but U is a closed set.

$\therefore$ $\boxed{\text{No.}}$

Is the set $\{(x,y,z) \in \mathbb{R}^3 \mid z=0,\ x^2+y^2 < 1\}$ a reg. surf?

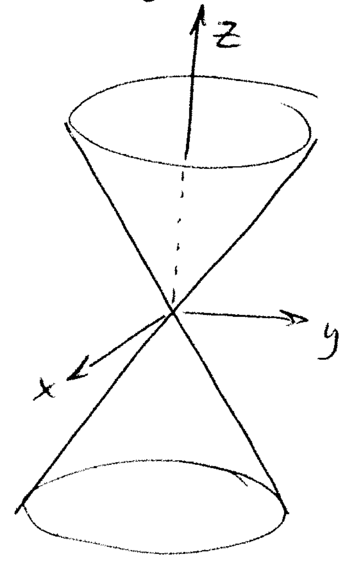
Same approach yields $\boxed{\text{Yes}}$.

Poblem 3

Show that the two sheeted cone is <u>not</u> a regular surface

$z^2 = x^2+y^2 \qquad (x,y) \in \mathbb{R}^2$

Use proposition 3.



Problem 4

Use definition 2. $f(x,y,z) = z^2$

$f(x,y,z) : \mathbb{R}^3 \longrightarrow \mathbb{R}^1$ $U = \mathbb{R}^3$

$$df = \begin{pmatrix} 0 \\ 0 \\ 2z \end{pmatrix}$$

when $z = 0$, $df = 0$. $\therefore$ df is not onto

$\therefore$ z is a critical value

?

However $f^{-1}(0)$ = the x-y plane $= \mathbb{R}^2 = f^{-1}(x,y,z) = (x,y$
(I think) If this is right, the $f^{-1}(0)$ is a regular surface since $\mathbb{R}^2$ can be parametrized by its natural coordinates